

MATH 1501 SUPPLEMENTARY HOMEWORK PROBLEMS

- (1) Use the definition of the derivative to find the following derivatives:
- (a) $f(x) = x^2$
 - (b) $g(x) = x^2|x|$ (Hint: Consider the cases $x < 0$ and $x \geq 0$ separately.)
 - (c) $m(x) = \sqrt{x^2 + 1}$
 - (d) $k(x) = \frac{1}{\sqrt{x^3 + 1}}$
 - (e) $F(x) = f(ax)$ where a is a constant and f is a differentiable function.
- (2) Compute the following derivatives using the rules for differentiation we learned in class (product rule, quotient rule, power rule, chain rule, etc.). You do not need to do any simplification of your answers!
- (a) $\frac{d}{d\theta} \left(\cos \left(\frac{\tan^2 \theta + \theta^2}{1 + \theta^2} \right) \right)$
 - (b) $\frac{d}{dx} \sqrt[2]{\sqrt[3]{\sqrt[4]{x^4 + 3x^2 + 2}}}$
 - (c) $\frac{d}{dt} \frac{F(t)G(t)H(F(t))}{\cos^2(\sqrt{t})}$ where F , G , and H are differentiable functions.
 - (d) $\frac{d}{dt} \frac{(at^2 + 1)^{4/5}}{\cot(\sin(bt^4))}$ where a, b are nonzero constants.
 - (e) $\frac{d}{du} \left(\frac{4u^3 + 3u - 6}{u^2 + 4} + \frac{d}{du} \sin(\sqrt[3]{u}) \right)$
 - (f) $\frac{d}{dx} \frac{\sqrt{2x + 3} \sqrt[3]{5x^2 - 4} \sin^2(3x)}{\csc^3(x)(x^2 + 3x + 3)^4}$
 - (g) $\frac{d}{dx} \sqrt[4]{x + \sqrt[3]{\sin(x) + \sqrt{\tan(x^2) + 3x}}}$
 - (h) $\frac{d}{dx} \sqrt{\frac{2x \tan^3(x)}{x^3 - 1}}$
- (3) Find $f''(x)$ if $f(x) = \cos(\sin(8x))$.
- (4) Find the tangent line to $xy^2 = \sin(x + y)$ at the point $(0, \pi)$.
- (5) Consider the curve defined by the equation $y^2 + x^3 = 3xy$. Find all points where the slope of the tangent line is 0. You may ignore the point $(0, 0)$.
- (6) Find the tangent line to the curve defined by $y^3 - 2x^2y = \sin(3x + 2y)$ at the point $(\pi, 0)$.
- (7) Find the maximum and minimum values of the following functions on the specified intervals

- (a) $f(x) = x^{1/3} + (2-x)^{1/3}$ on the interval $[-1, 2]$
 (b) $g(x) = a \sin(x) + \cos(x)$ on $[0, \frac{\pi}{2}]$, $a \geq 0$
- (8) Show that if $0 < |a| < |b|$, then the equation $bx = \cos(ax)$ has at most one real solution.
- (9) Show that, regardless of the value of b , there is at most one point x in the interval $-1 \leq x \leq 1$ for which $x^3 - 3x + b = 0$.
- (10) Let $f(x) = 1 - x^{2/3}$. Show that $f(1) = f(-1) = 0$, but that $f'(x)$ is never zero in the interval $[-1, 1]$. Explain how this is possible, in view of Rolle's Theorem.
- (11) Define a function f as

$$f(x) = \begin{cases} \frac{3-x^2}{2} & \text{if } x \leq 1; \\ \frac{1}{x} & \text{if } x > 1. \end{cases}$$

Show that f satisfies the conditions of the Mean Value Theorem over the interval $[0, 2]$ and determine all the values of c in $[0, 2]$ that satisfy its conclusion.

- (12) The radius of a right circular cylinder increases at a constant rate. Its height is a linear function of the radius and increases three times as fast as the radius. When the radius is 1 foot the altitude is 6 feet. When the radius is 6 feet, the volume is increasing at a rate of 1 cubic foot per second. When the radius is 36 feet, the volume is increasing at a rate of n cubic feet per second, where n is an integer. Compute n .
- (13) A propane tank is to be made of a cylinder of radius r and length h surmounted by two hemispheres of radius r . (One hemisphere is placed on each end of the cylinder.) Suppose the volume is to be a fixed value V . Find the value of h that minimizes the surface area of the tank.
- (14) Among all right circular cylinders of given lateral surface area, prove that the smallest circumscribed sphere has radius $\sqrt{2}$ times that of the cylinder.
- (15) Sketch the graph of a function satisfying
- f is differentiable for all x except $x = 1$.
 - $f(-1) = f(\frac{1}{2}) = f(2) = 0$.
 - f is increasing on $(0, 1)$ and $(1, 3)$.
 - f is decreasing on $(-\infty, 0)$ and $(3, \infty)$.
 - f is concave upwards on $(-\infty, 1)$ and on $(4, \infty)$.
 - f is concave downwards on $(1, 4)$.
 - f has a local minimum at $x = 0$.
 - f has a local maximum at $x = 3$.
 - f has an inflection point at $x = 4$.
 - $\lim_{x \rightarrow 1^-} f(x) = +\infty$.
 - $\lim_{x \rightarrow 1^+} f(x) = -\infty$.
 - $\lim_{x \rightarrow \infty} f(x) = 1$.

- (16) Sketch the graph of a function satisfying
- (a) f is differentiable for all x except $x = -2$ and $x = 0$.
 - (b) $f(-5) = f(-3) = f(-1) = f(1) = f(3) = 0$.
 - (c) f is increasing on $(-\infty, -4)$, $(-2, 0)$, and $(0, 2)$.
 - (d) f is decreasing on $(-4, -2)$ and $(2, \infty)$.
 - (e) f is concave upwards on $(-\infty, -5)$ and on $(-1, 0)$.
 - (f) f is concave downwards on $(-5, -2)$, $(-2, -1)$, and $(0, \infty)$.
 - (g) f has a local maximum at $x = -4$ and at $x = 2$.
 - (h) f has no local minima.
 - (i) f has an inflection point at $x = -5$ and at $x = -1$.
 - (j) $\lim_{x \rightarrow -2^+} f(x) = -\infty$.
 - (k) $\lim_{x \rightarrow -2^-} f(x) = -\infty$.
 - (l) $\lim_{x \rightarrow 0^-} f(x) = +\infty$.
 - (m) $\lim_{x \rightarrow 0^+} f(x) = -\infty$.
 - (n) $\lim_{x \rightarrow \infty} f(x) = -1$.
- (17) Sketch the graph of $f(x) = (x^3 - 3x)^{\frac{1}{3}}$.
- (18) Sketch the graph of $g(x) = \frac{x^3 - 2x + 1}{x - 1} = x - \frac{x - 1}{x^2 - 1}$.