School of Mathematics
Georgia Tech

Math 1502, QUIZ 3

Date: January 28, 2008 Name (printed; last name first) and section: KEY

There are two problems on this quiz, each worth four points. Two extra points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

(1) Is the series $\sum_{n=1}^{\infty} \frac{\ln \sqrt{n}}{n}$ convergent? Justify!

Rewrite the series as $\sum_{n=1}^{\infty} \frac{\ln\sqrt{n}}{n} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{\ln n}{n}$. Thus, our series has the same convergence properties as $\sum_{n=1}^{\infty} \frac{\ln n}{n}$. There are two ways to see this diverges:

(1) Integral test: $\int_1^{\infty} \frac{\ln x}{x}\,dx = \lim_{b\to\infty} \int_1^b \frac{\ln x}{x}\,dx = \lim_{b\to\infty} \frac{(\ln x)^2}{2}\Big|_1^b = \infty$

Since the integral diverges, the series diverges.

(2) Basic comparison: For $n \geq 3$, $\frac{\ln n}{n} \geq \frac{1}{n}$, so since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, our series diverges.

(2) Is the series $\sum_{k=1}^{\infty} \frac{k2^k}{3^k}$ convergent? Justify!

Consider the ratio test:

$$\frac{\frac{(k+1)2^{k+1}}{3^{k+1}}}{\frac{k2^k}{3^k}} = \left(\frac{k+1}{k}\right)\cdot\frac{2}{3} \xrightarrow[k\to\infty]{} \frac{2}{3} < 1$$

Thus, the series converges by the ratio test.