School of Mathematics
Georgia Tech

Math 1502, QUIZ 4

Date: February 06, 2008 Name (printed; last name first) and section: SOLUTION

There is one problem on this quiz that is worth eight points. Two extra points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

Write the Taylor Series expansion of $f(x) = \ln(1+2x)$ in powers of $x-1$.

We know that $\ln(1+u) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} u^k$, so need to rewrite $\ln(1+2x)$ to use this but with $x-1$.

$$\begin{aligned}
\ln(1+2x) &= \ln(1+2x-2+2) \\
&= \ln(3+2(x-1)) \\
&= \ln\left(3\cdot\left(1+\tfrac{2}{3}(x-1)\right)\right) \\
&= \ln 3 + \ln\left(1+\tfrac{2}{3}(x-1)\right) \\
&= \ln 3 + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}\left(\tfrac{2}{3}(x-1)\right)^k = \boxed{\ln 3 + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}2^k}{k3^k}(x-1)^k}
\end{aligned}$$

Or, we can do by definition:

$f(1) = \ln 3$

$f'(x) = \frac{2}{1+2x}$

$f''(x) = -\frac{2^2}{(1+2x)^2}$

$f^{(3)}(x) = \frac{2\cdot 2^3}{(1+2x)^3}$

$f^{(4)}(x) = \frac{-6\cdot 2^4}{(1+2x)^4}$

$f^{(5)}(x) = \frac{24\cdot 2^5}{(1+2x)^5}$

Suggest: $f^{(n)}(x) = \frac{(-1)^{n+1}(n-1)!\,2^n}{(1+2x)^n}$

$f^{(n)}(1) = (-1)^{n+1}(n-1)!\left(\frac{2}{3}\right)^n$

Therefore, the series is

$$\ln(1+2x) = \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}\cdot(n-1)!\,\frac{2^n}{3^n}(x-1)^n$$

$$= \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\frac{2^n}{3^n}(x-1)^n$$