

Math 1502, QUIZ 5

Date: February 13, 2008

Name (printed; last name first) and section: SOLUTION

There is one problem on this quiz that is worth eight points. Two extra points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

Find the radius and the interval of convergence for the series  $\sum_{k=1}^{\infty} \frac{\ln k}{2^k} (x-2)^k$ .

We check the convergence properties of

$$\sum_{k=1}^{\infty} \left| \frac{\ln k}{2^k} (x-2)^k \right| = \sum_{k=1}^{\infty} \frac{\ln k}{2^k} |x-2|^k$$

by the ratio test:

$$\frac{\frac{\ln(k+1)}{2^{k+1}} |x-2|^{k+1}}{\frac{\ln(k)}{2^k} |x-2|^k} = \frac{\ln(k+1)}{2 \ln(k)} |x-2| \xrightarrow{k \rightarrow \infty} \frac{|x-2|}{2}$$

For the series to converge, we need  $\frac{|x-2|}{2} < 1 \Leftrightarrow |x-2| < 2$

$$\Leftrightarrow -2 < x-2 < 2 \Leftrightarrow 0 < x < 4.$$

Thus, the radius of convergence is 2. For the interval,

we check the endpoints:

$$x=0 \quad \sum_{k=1}^{\infty} \frac{\ln k}{2^k} (0-2)^k = \sum_{k=1}^{\infty} (-1)^k \ln k \quad \text{diverges, since } \ln k \xrightarrow{k \rightarrow \infty} \infty.$$

$$x=4 \quad \sum_{k=1}^{\infty} \frac{\ln k}{2^k} (4-2)^k = \sum_{k=1}^{\infty} \ln k \quad \text{diverges, since } \ln k \xrightarrow{k \rightarrow \infty} \infty$$

$$\boxed{\text{Interval of convergence: } (0, 4)}$$