School of Mathematics
Georgia Tech

Math 1502, QUIZ 6

Date: February 20, 2008 Name (printed; last name first) and section: SOLUTION

There is one problem on this quiz that is worth eight points. Two points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

Consider a *linear transformation* $f : \mathbb{R}^3 \to \mathbb{R}^2$ such that

$$f(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad f(\mathbf{e}_2) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad f\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

(*a*) (5 points) Write down the matrix of the transformation f;
Hint: One needs $f(\mathbf{e}_3)$ as well! That can be found by using the equations in the display above and the linearity of f.

Note that $\vec{e}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \vec{e}_1 - \vec{e}_2$. Since f is linear, $f(\vec{e}_3) = f\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) - f(\vec{e}_1) - f(\vec{e}_2) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
$= \begin{bmatrix} -4 \\ 1 \end{bmatrix}$. The matrix of f, A_f, is

$$[f(\vec{e}_1) \; f(\vec{e}_2) \; f(\vec{e}_3)] = \begin{bmatrix} 2 & 1 & -4 \\ 1 & -1 & 1 \end{bmatrix}$$

(*b*) (3 points) Compute $f(\mathbf{v})$, where

$$\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$

$\vec{v} = 2\vec{e}_1 + \vec{e}_2 - \vec{e}_3$, so

$$f(\vec{v}) = 2f(\vec{e}_1) + f(\vec{e}_2) - f(\vec{e}_3)$$
$$= 2\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$
$$= \boxed{\begin{bmatrix} 9 \\ 0 \end{bmatrix}}$$