

Math 1502, QUIZ 8

Date: March 12, 2008 Name (printed; last name first) and section: SOLUTION

There is one problem on this quiz that is worth eight points. Two points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

Consider the matrix and the vector

$$A = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ -2 & -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(a) (6 points) Perform the LU-factorization for A;

$$\left[ \begin{array}{cc|cc} 1 & -2 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ -2 & -1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & -5 & 2 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -3 & -5 & 1 \end{array} \right]$$

$\begin{matrix} u & R \end{matrix}$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -3 & -5 & 1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & -5 & 1 & 3 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 5 & 1 \end{array} \right]$$

$\begin{matrix} L \end{matrix}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

(b) (2 points) Is  $A\mathbf{x} = \mathbf{b}$  solvable?

Rank(A) = 2, so look at the matrix C of the last 3-2 = 1 row of R:  $C = [-3 \ -5 \ 1]$

$A\mathbf{x} = \mathbf{b}$  is solvable if and only if  $C\mathbf{b} = 0$ :

$$[-3 \ -5 \ 1] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = -3 - 5 + 1 = -7 \neq 0,$$

so  $A\mathbf{x} = \mathbf{b}$  is not solvable.