School of Mathematics
Georgia Tech

Math 1502, QUIZ 10

Date: April 02, 2008 Name (printed; last name first) and section: SOLUTION

There is one problem on this quiz that is worth eight points. Two points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

Consider the subspace $S \subset \mathbb{R}^3$ spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Find the *orthogonal projection* matrices onto S and $S^\perp$.

Let $V = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}$. Then the orthogonal projection matrix onto S, denoted P, is given by

$$P = V(V^tV)^{-1}V^t.$$

$$V^tV = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 9 & 14 \end{bmatrix}$$

$$(V^tV)^{-1} = \frac{1}{6\cdot 14 - 9\cdot 9}\begin{bmatrix} 14 & -9 \\ -9 & 6 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 14 & -9 \\ -9 & 6 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}\left(\frac{1}{3}\begin{bmatrix} 14 & -9 \\ -9 & 6 \end{bmatrix}\right)\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 5 & -3 \\ -4 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Now the orthogonal projection matrix onto $S^\perp$, denoted $P^\perp$, is

$$P^\perp = I - P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$