HOMEWORK 2

Problem 1: Suppose that A_1 and A_2 are two $m \times n$ matrices and that $A_1\vec{x} = A_2\vec{x}$ for every vector $\vec{x} \in \mathbb{R}^n$. Prove that $A_1 = A_2$.

Problem 2: Let A be an invertible $n \times n$ matrix and assume that it can be row reduced without row swaps. One can then write

$$A = LDU$$

where L is lower triangular with all diagonal elements equal to 1, U is upper triangular with all diagonal elements equal to 1 and D diagonal consisting of the non-zero pivots. Show that the above factorization is unique, i.e., if

$$A = L'D'U'$$

is another such factorization, then $L = L', D = D'$ and $U = U'$.

Problem 3: a) Suppose that $A = A^T$ can be row reduced without row swaps. If E is an elementary matrix such that EA has zero as a second entry in the first column, what can you say about EAE^T.

b) Use step a) to prove that any symmetric matrix that can be row reduced without swaps can be written as

$$A = LDL^T$$

c) How should one change the statement if swaps are needed?

Please work problems 11, 25, 28 in Section 2.1 and problems 19, 24, 27 in section 2.2.

Please turn it in for grading on Thursday Januar 23.