

## PUZZLES AND GAMES

## PROBLEMS (DUE SEPTEMBER 18):

1. You have an  $m \times n$  rectangular chocolate bar made of  $mn$  small squares, and you wish to break it up into its constituent squares. At each step, you may pick up one piece and break it along one of its vertical or horizontal grooves. Prove that every method finishes in the same number of steps.
2. Two players play a game with a  $6 \times 10$  bar of chocolate made of 60 small squares. A move consists of breaking the bar along any of its marked horizontal or vertical grooves into two smaller rectangular bars, and eating one of the two rectangular pieces. The players alternate moves, and the game continues until one piece is left. The person ending up with the single piece is the loser. Which player has a winning strategy in this game, and what is it?
3. A queen is placed randomly on a standard  $8 \times 8$  chessboard, and then two players take turns moving the queen any number of squares to the right, upwards, or along a diagonal to the right and upwards. The player who manages to place the queen in the upper-right hand square wins. What is the probability that the second player has a winning strategy?
4. You play a game of solitaire with a given  $m \times n$  array of real numbers. A move in the game consists of reversing the signs of all the numbers in any row or column, and the goal is to end up with all row sums and all column sums nonnegative. Prove that it is always possible to win the game.
5. A school playground game is played as follows. Children are arranged clockwise around a circle in positions labeled 1 through  $n$ . The child in position 2 steps out of the circle, and continuing clockwise, every second child among those who remain steps out until only one child is left. Let  $f(n)$  be the position of the final remaining child. (Thus, for  $n = 5$ , the children are eliminated in the order 2, 4, 1, 5, 3, and  $f(5) = 3$ .) Find a formula for  $f(n)$  in terms of the binary expansion of  $n$ .

6. An integer  $n$ , unknown to you, has been randomly chosen in the interval  $[1, 2008]$  with uniform probability. Your objective is to select  $n$  in an *odd* number of guesses. After each incorrect guess, you are informed whether  $n$  is higher or lower, and you must guess an integer on your next turn among the numbers that are still feasibly correct. Find a strategy with which the probability of winning is greater than  $2/3$ .