Homework Assignment # 4
Due Thursday, September 20

1. Apostol §3.10, p. 103, Exercise # 24deh

2. Let V denote the linear space consisting of all polynomials whose third derivative is zero. What is the dimension of V?

3. Determine whether or not the set
$$S = \{(1,1,0,0,0)\,(-1,1,1,0,0),\ (0,4,2,0,0)\}$$
of vectors in $\mathbf{R}^5$ is linearly independent. What is dimension of the linear span of S?

4. Find a basis for the subspace W of $\mathbf{R}^4$ spanned by the vectors
$$(1,1,2,4),\ (2,-1,-5,2)\,(1,-1,-4,0)\,(2,1,1,6),$$
and determine the dimension of W.

5. Find a basis for the nullspace of the matrix
$$A = \begin{bmatrix} 1 & 2 & 1 & -3 \\ 2 & 4 & 4 & -1 \\ 3 & 6 & 7 & 1 \end{bmatrix}$$

6. Find the dimension and a basis for the subspace of $\mathbf{R}^5$ consisting of all solutions to the following system of homogeneous linear equations:
$$\begin{aligned} x + 2y - 4z + 3u - v &= 0 \\ x + 2y - 2z + 2u + v &= 0 \\ 2x + 4y - 2z + 3u + 4v &= 0 \end{aligned}$$

7. Find the coordinates of the vector $v = (3,1,-4) \in \mathbf{R}^3$ relative to the ordered basis $w_1 = (1,1,1), w_2 = (0,1,1), w_3 = (0,0,1)$ of $\mathbf{R}^3$.

8. Let P_2 be the linear space of all polynomials of degree at most 2.

   (a) Is $t^2 - 1$ in the subspace of P_2 spanned by $\{1+t, 1-t, t^2+2\}$?

   (b) Show that the polynomials $f_1 = 1, f_2 = t - 1, f_3 = (t-1)^2$ form a basis for P_2.

   (c) Find the coordinates of $2t^2 - 5t + 6$ relative to the ordered basis (f_1, f_2, f_3) for P_2.

9. Let V be a finite-dimensional linear space. It follows from Theorem 3.5 in Apostol that $\dim(V)$ is the maximum number of linearly independent vectors in V. Prove that $\dim(V)$ is also the *minimum* number of vectors which *span* V.

10. Apostol §3.13, p. 109, Exercise # 6

11. Apostol §3.13, p. 110, Exercise # 12

12. Apostol §3.13, p. 110, Exercise # 14ac