Homework Assignment #6

Due Thursday, October 4

1. Apostol §3.17, p.118, Exercise #8
2. Apostol §3.17, p.118, Exercise #9
3. Apostol §4.4, p.123, Exercise #5
4. Apostol §4.4, p.123, Exercise #7
5. Apostol §4.4, p.123, Exercise #11
6. Apostol §4.4, p.123, Exercise #12
7. Apostol §4.4, p.123, Exercise #23
8. Does there exist a linear transformation T from $\mathbf{R}^3$ to $\mathbf{R}^2$ such that

$$T\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix},\ T\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix},\ \text{ and } T\begin{bmatrix}1\\1\\-1\end{bmatrix} = \begin{bmatrix}-2\\-2\end{bmatrix}?$$

   (Be sure to justify your answer.)

9. Find the least squares regression line for the data points

$$(0,10), (2,6), (3,7), (4,6), (5,3), (8,1).$$

10. Let

$$A = \begin{pmatrix}2 & 1\\4 & 2\\1 & 1\end{pmatrix},\ b = \begin{bmatrix}1\\0\\0\end{bmatrix}.$$

   (a) Find the orthogonal projection of b onto the range of A.

   (b) Find the best approximate (least squares) solution to the overdetermined system of equations $Ax = b$.

11. Recall that *mean* $\overline{x}$ and *standard deviation* σ_x of a collection $x_1, \ldots, x_n$ of real numbers are given by the formulas

$$\overline{x} = \frac{\sum_{i=1}^n x_i}{n}, \ \sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \overline{x})^2}{n}}.$$

The *Pearson correlation coefficient* of a collection of $n \geq 2$ distinct data points $(x_1, y_1), \ldots, (x_n, y_n) \in \mathbf{R}^2$ is defined to be

$$r = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{n\sigma_x\sigma_y},$$

where σ_x (resp. σ_y) denotes the standard deviation of $x_1, \ldots, x_n$ (resp. $y_1, \ldots, y_n$) and $\overline{x}$ (resp. $\overline{y}$) denotes the mean of $x_1, \ldots, x_n$ (resp. $y_1, \ldots, y_n$). (The quantity $\frac{1}{n}\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})$ is called the *covariance*, so the Pearson correlation coefficient is just the covariance divided by the product of the standard deviations.)

(a) Prove that $-1 \leq r \leq 1$, and that the data points lie on a straight line if and only if $r = \pm 1$. [**Hint:** Use the Cauchy-Schwartz inequality.]

(b) Calculate the Pearson correlation coefficient for the data points in Problem 9.

12. Let V be a finite-dimensional Euclidean space, let W be a subspace of V, and let p_W denote orthogonal projection onto W. Prove that for every $x \in V$, the length of $p_W(x)$ is less than or equal to the length of x, with equality if and only if $x \in W$.