Half Hour Quiz Number1 Answers

■ Problem 1

Find the Taylor derivatives of ln(2 + x)

```
In[94]:= f[x_] := Log[2 + x]
```

Compute derivatives:

```
In[92]:= Table[
          {n, D[Log[2 + x], {x, n}]},
          {n, 0, 10}] // TableForm
```

0	Log[2 + x]
1	$\frac{1}{2+x}$
2	$-\frac{1}{(2+x)^2}$
3	$\frac{2}{(2+x)^3}$
4	$-\frac{6}{(2+x)^4}$
5	$\frac{24}{(2+x)^5}$
6	$-\frac{120}{(2+x)^6}$
7	$\frac{720}{(2+x)^7}$
8	$-\frac{5040}{(2+x)^8}$
9	$\frac{40320}{(2+x)^9}$
10	$-\frac{362880}{(2+x)^{10}}$

```
In[95]:= Table[
          {n, D[Log[2 + x], {x, n}] /. x -> 0},
          {n, 0, 10}] // TableForm
```

Out[95]//TableForm=

0	Log[2]
1	$\frac{1}{2}$
2	$-\frac{1}{4}$
3	$\frac{1}{4}$
4	$-\frac{3}{8}$
5	$\frac{3}{4}$
6	$-\frac{15}{8}$
7	$\frac{45}{8}$
8	$-\frac{315}{16}$
9	$\frac{315}{4}$
10	$-\frac{2835}{8}$

in general the answer (for n not zero) is:

In[93]:= **(−1)^(*n* + 1) (*n* − 1)! / (2 + *x*)^*n***

Out[93]= $(-1)^{1+n}\ (2+x)^{-n}\ (-1+n)!$

Calculate Series

```
In[96]:= Series[f[x], {x, 0, 3}]
```

Out[96]= $\text{Log}[2] + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24} + O[x]^4$

```
In[97]:= Series[f[x], {x, 0, 4}]
```

Out[97]= $\text{Log}[2] + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24} - \frac{x^4}{64} + O[x]^5$

```
In[98]:= Series[f[x], {x, 0, 5}]
```

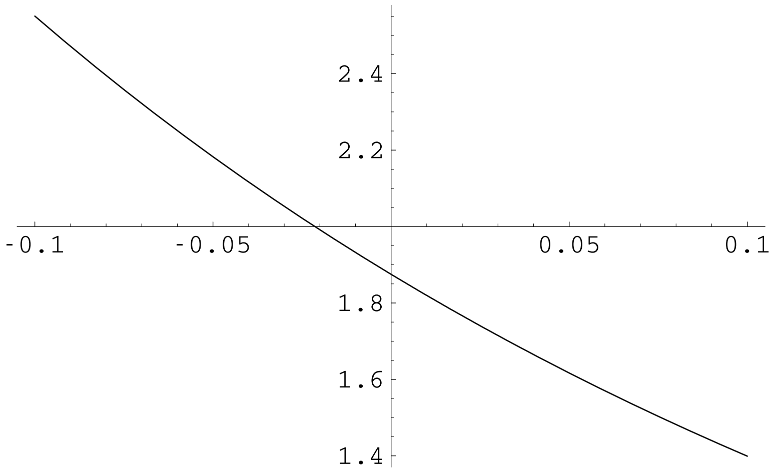
Out[98]= $\text{Log}[2] + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24} - \frac{x^4}{64} + \frac{x^5}{160} + O[x]^6$

```
In[101]:= fp = D[f[x], {x, 6}]
```

Out[101]= $-\frac{120}{(2+x)^6}$

The biggest this gets is at x = −1.

```
In[102]:= Plot[Abs[fp], {x, -.1, .1}]
```




```
Out[102]= - Graphics -

In[103]:= M = Abs[fp] /. x -> -.1

Out[103]= 2.5507

In[105]:= ErrorBound = M x^6 / 6!
```

Out[105]= $0.00354264\, x^{6}$

■ Problem 2

```
In[106]:= Limit[(E^x - (1 + x + x^2 / 2)) / (x^3 Cos[x]),
                x -> 0]
```

Out[106]= $\frac{1}{6}$

```
In[108]:= mun = Series[E^x - (1 + x + x^2 / 2), {x, 0, 3}]
```

Out[108]= $\frac{x^{3}}{6} + O[x]^{4}$

```
In[109]:= dem = Series[x^3 Cos[x], {x, 0, 4}]
```

Out[109]= $x^{3} + O[x]^{5}$

Evedently, the limit is 1 / 6 divided by 1.