

Georgia Institute of Technology
High School Mathematics Competition 2006
Junior Varsity Proof-Based Test
Problem #1

ID#:

The positive real numbers x, y satisfy $x+y = 2$. Prove that $xy(x^2+y^2) \leq 2$

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Problem #2

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Among all pairs of $2k$ digit natural numbers a, b (in decimal representation) that have the same set of digits with all digits positive, what is the largest possible value of $a - b$? Prove your results. For example, if $n = 2$ then the largest value is $91 - 19 = 72$.

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Problem #3

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In a marching band practice, the members are arranged in a rectangular formation. Looking at the people in this formation, the director notes that among those who are the shortest in their respective columns, Al is the tallest. Also, among those who are the tallest in their respective rows, Bob is the shortest. Assuming that no 2 people have the same height, is Al taller than Bob?

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Problem #4

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A popular game in Asia is to take four numbers and to try combining them into the number 24 using only $+$, $-$, \times , \div . Each number must be used once and only once. For example, with the four numbers 2, 4, 5, 8 we can get 24 in a number of ways:

$$(4 \times 5) + (8 \div 2) = 24, \text{ or } (2 + 5 - 4) \times 8 = 24, \text{ or } 2 \times 4 \times (8 - 5) = 24.$$

Can you make 24 using the numbers 5, 5, 5, 1?

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Problem #5

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Consider a square and a rhombus inscribed in the square with its vertices on the boundary of the square. Prove that the rhombus is in fact a square.