

Georgia Institute of Technology  
High School Mathematics Competition 2006

Varsity Proof-Based Test  
Problem #1

ID#:

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The roots of  $16(1 - x) = \frac{1}{x}$  are  $x_1, x_2$ . The tenth digit after the decimal point of  $x_1$  is 8; find the tenth such digit in  $x_2$ .

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Problem #2

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Among all pairs of  $2k$  digit natural numbers  $a, b$  (in decimal representation) that have the same set of digits with all digits positive, what is the largest possible value of  $a - b$ ? Prove your results. For example, if  $n = 2$  then the largest value is  $91 - 19 = 72$ .

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Problem #3

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Prove that, for integers  $m \geq 1$ ,  $(4m)!(m!)^4 > ((2m)!)^4$ .

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Problem #4

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2006 can be factored as  $2 \times 17 \times 59$ . Prove that for all  $x \in \mathbb{R}$

$$\left(\frac{1003}{2}\right)^x + \left(\frac{118}{17}\right)^x + \left(\frac{34}{59}\right)^x \geq 2^x + 17^x + 59^x.$$

Remark:  $2006 = 1003 \cdot 2 = 118 \cdot 17 = 34 \cdot 59$ .

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Problem #5

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In  $\triangle ABC$ ,  $\angle ABC = 30^\circ$ , and  $\angle ACB = 45^\circ$ . Let  $E$  be inside of this triangle in such a way that triangle  $EBC$  is isosceles (i.e.  $|EB| = |EC|$ ), and  $\angle EBC = 15^\circ$ . Denote by  $F$  the intersection of  $CE$  and  $AB$ . Show that  $|AF| = |FB|$ .

