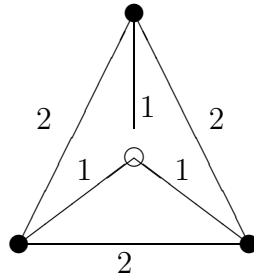


CS 6550 - Design and Analysis of Algorithms - Fall 2007

Homework 1

1. Consider the following algorithm for Vertex Cover: Run DFS. Output the nodes which are not leaves in the DFS tree. Show that the output is indeed a vertex cover, and that this algorithm gives yet another 2-approximation for the minimum vertex cover.
2. We claimed in class that a bad approach to approximating the optimal vertex cover is to use a greedy method that does the following: pick the vertex that covers the most yet uncovered vertices, add it to the cover, and repeat. Give an infinite family of examples (i.e., one for each value of n) that shows that this method will not achieve an approximation ratio of 2.
3. We will consider the *Steiner Minimum Tree* (SMT) problem. We are given a graph $G = (V, E)$ with weight function w , and a set of vertices $V' \subseteq V$ of *terminal nodes*. We want to find a minimum-cost tree $T \subseteq G$ that spans the vertices of V' . The tree T may use nodes in $V - V'$. For instance, consider the graph, where the dark vertices are in V' and the middle vertex is not:



Notice that the minimum tree for connecting the dark vertices in V' without using other vertices has total weight 4, but using the middle vertex we can achieve this with weight 3. The tree that achieves the minimum weight (possibly using vertices in $V - V'$) is called the *Steiner tree* and any vertex in T but not in V' is called a *Steiner node*. The SMT problem takes input $G = (V, E), V' \subseteq V, C$ and asks whether or not there is a Steiner tree of weight at most C .

Let us suppose our graph satisfies the triangle inequality: $w(x, y) + w(y, z) \geq w(x, z)$ for all $x, y, z \in V$. Let T' be the minimum spanning tree on V' . Show that T' is a 2-approximation for the SMT problem (i.e., if T is a minimum SMT, then $w(T') \leq 2w(T)$). there is a Steiner tree

4. Find a polynomial-time $4/3$ -approximation to instances of metric TSP where distances are either 1 or 2.

Hint: The 2-matching problem (i.e., find a minimum weight subset of edges M such that each node is adjacent to *exactly* 2 edges in M) can be solved in polynomial time.

5. The following problem arises in telecommunications networks, and is known as the SONET ring loading problem. The network consists of a cycle on n nodes, numbered 0 through $n - 1$ clockwise around the cycle. Some set C of calls is given; each call is a pair (i, j) originating at node i and ending at node j . The call can be routed either clockwise or counterclockwise around the ring. The objective is to route the calls so as to minimize the total load on the network. The load L_i on link $(i, i + 1 \pmod n)$ is the number calls routed through the link, and the total load is $\max_{1 \leq i \leq n} L_i$. Give a 2-approximation algorithm for the SONET ring loading problem.