

CS 3511 - Spring 2009
Homework 1
Due: February 11

The honors students must do this homework in addition to the standard (3510) homework. Hopefully these will be a little more challenging and interesting. Non-honors students are also encouraged to do these problems! **The “extra-credit” applies to everyone!** Again, you need to work alone and without calculators.

1. Compute $x^y \bmod 23$ for $x = 13$ and $y = 37^{192}$.
2. Prove the Master Theorem.
 - (a) First, argue that if at each level a problem of size x is broken into a problems of size x/b and if it takes x^d work to recombine the results from the recursion, then:
 - i. The top level of the tree requires n^d work.
 - ii. The bottom level of the tree has $a^{\log_b n} = n^{\log_b a}$ leaves, and requires that much total work.
 - iii. Figure out how much work each intermediate level of the tree requires, and write the *total* amount of work (on all levels of the tree) as a summation.
 - (b) Now evaluate your summation in three cases: (a) $d = \log_b a$, (b) $d < \log_b a$, (c) $d > \log_b a$.

Hint: For parts (b) and (c), it will be useful for you to verify and use the following identity for appropriate choices of x, r and k :

$$x + xr + xr^2 + \dots + xr^k = \frac{xr^{k+1} - 1}{r - 1}.$$

3. **Extra Credit:** Seven prisoners in the land of Mathtopia are presented with a one-time chance to gain their freedom, but if they fail then they will all be killed. At dawn, an integer between 1 and 7 will be written on each of their foreheads. Each sees the numbers on the other prisoners' heads, but cannot see his own. They simultaneously have to guess the number on their own head, and no communication between them prior to this is allowed. If even one person guesses correctly, then they will all be saved. However, if they are all incorrect, then well, you get the idea.

Devise a strategy that will ensure their survival, no matter what numbers are placed on their heads!!