CS 1050 - Proofs
Homework 3
Assigned September2
Due <u>Thursday</u>, September 9

1. Let $f : S \rightarrow T$ and $g : T \rightarrow U$ be two functions such that $g \circ f$ is one-to-one. Prove that f is one-to-one. Give an example for which $g \circ f$ is one-to-one but g is not one-to-one.

2. Let $f : S \rightarrow T$ and $g : T \rightarrow U$ be two functions such that $g \circ f$ is onto U. Prove that g is onto U. Give an example for which $g \circ f$ is onto U but f is not onto T.

3.a) Let $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ by $f(x_1, x_2) = (2x_1 + x_2, 3x_1 - x_2, 2x_1 + x_2)$ for all reals x_1, x_2.

Prove that f is one-to-one.

b) For the function given in part (a), *disprove* the following conjecture:

Conjecture 1 *f is onto.*

4. Show thhe following theorems:

a) If $A \cup B \subseteq A \cap B$ then $A = B$.

b) $(A \cap \emptyset) \cup B = B$.

5. a) We know that

Theorem 1 *If A, B are subsets of a universe U, then $(A \cup B)^c = A^c \cap B^c$.*

Use this fact to prove the following theorem:

Theorem 2 *If A, B, C are subsets of U, then $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$.*

Do not prove Theorem 2 by taking an arbitrary element x of the left-hand side, and showing it is in the RHS, and then showing that an arbitrary element y of the RHS is in the LHS. Do it by cleverly applying Theorem 1.