1. Prove that for all integers $n \geq 1$, $4 + 10 + 16 + \ldots + (6n - 2) = n(3n + 1)$.

2. Prove (using induction) that $8^n - 2^n$ is a multiple of 6 for every integer $n \geq 1$.

3. Prove that $5^n - 4n - 1$ is divisible by 16 for all integers $n \geq 1$.

4. a) Prove the following lemma:

   Lemma 1 *For all* reals numbers $x \geq 4$, $(x + 1)^2 \leq 2x^2$.

   b) Now use it to prove the following theorem:

   Theorem 2 *For all integers* $n \geq 4$, $n^2 \leq 2^n$.

5. What amounts of money can you make using just dimes and quarters? Prove your answer using mathematical induction.

6. What amounts of postage can you make with 5 and 6 cent stamps? Prove your answer using induction.

7. Show that $1^2 + 2^2 + \ldots + n^2 = n(n + 1)(2n + 1)/6$ for all positive integers n.