CS 1050 - Proofs
Homework 9
Assigned Thursday, October 28
Due Thursday, November 4

1. Let $f(n) = 2^{n+1}$ for all $n \geq 1$. Let $g(n) = 2^n$ for all $n \geq 1$. Without using a limit, prove that $f = O(g)$.

2. a) Let $f(n) = (n+1)^2$ for all $n \geq 1$. Let $g(n) = n^2$ for all $n \geq 1$. Using a limit, prove that $f = O(g)$.

b) Now prove the same theorem without a limit.

3. Let $f(n) = (200n)^2 + 50$ for all $n \geq 1$. Let $g(n) = n^3$ for all $n \geq 1$. With or without a limit, prove that $f = O(g)$.

4. Let $f(n) = n^2$ and let $g(n) = 4n^2 + 5n - 6$. Prove that $f = O(g)$.

5. Prove the following theorem.

Theorem 1 *For any positive number A, there is a number N (which of course can depend on A) so that for all $n \geq N$, $A^n < n!$.*

6. Let $f(n) = n^2$ if $n \geq 1$ and n is even and $f(n) = 1$ if $n \geq 1$ and n is odd. Let $g(n) = n$ for all $n \geq 1$.

a) Prove that f is *not* $O(g)$. Warning: You cannot use a limit (why?). Make sure you show that you understand what it means for one function *not* to be big-O of another one before trying to prove that f is not $O(g)$. (Think about what you would need to prove. Given *any* c, n_0, you want to show that you can find an $n \geq n_0$ so that $f(n) > cg(n)$. Do you see why this is what you want?)

b) Prove that g is *not* $O(g)$.