Throughout, A is a commutative ring with $0 \neq 1$.

1. Let k be a field, and $k[x]$ a polynomial ring. Prove that $k[x]$ contains infinitely many distinct monic irreducible polynomials.
2. Let $S \subset \mathbb{Z}$ be the multiplicative set consisting of all positive odd integers. What are all ideals of $S^{-1}\mathbb{Z}$?
3. If A is a domain which is not a field, prove that $A[x]$ is not a principal ideal domain.
4. If every submodule of every free A-module is free, prove that A is a principal ideal domain.
5. In which of the following rings is every ideal principal? Justify your answer.
$$(i)\ \mathbb{Z} \oplus \mathbb{Z}, \qquad (ii)\ \frac{\mathbb{Z}}{(4)}, \qquad (iii)\ \frac{\mathbb{Z}}{(6)}[x], \qquad (iv)\ \frac{\mathbb{Z}}{(4)}[x].$$
6. Let $f(x) \in \mathbb{Z}[x]$ be a monic polynomial. If $f(a) = 0$ for some $a \in \mathbb{Q}$, prove that $a \in \mathbb{Z}$.
7. Let M be a 3×3 matrix with complex entries. If M^3 is the identity matrix, what are the possibilities for the Jordan canonical form of M?
8. Let M be a 3×3 matrix with integer entries and $\det(M) = -1$. Assume that every real eigenvalue of M is rational. What are the possibilities for the minimal polynomial and Jordan canonical form of M?
9. Let G be a finitely generated Abelian group presented by the following matrix:
$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$
(Recall that this means that $G \cong$ co-kernel(ϕ) where $\phi : \mathbb{Z}^4 \to \mathbb{Z}^3$ is the linear map whose matrix with respect to the standard bases is A.) Express G as a direct sum of cyclic groups.
10. Complete the course evaluation form available online at: www.coursesurvey.gatech.edu.