

**HW 19:** Suppose we wish to solve this problem by the revised simplex algorithm:

$$\begin{aligned} &\max x_3 \text{ subject to} \\ &-3x_1 - x_2 + x_3 \leq 0 \\ &-2x_1 - 5x_2 + x_3 \leq 0 \\ &x_1 + x_2 = 1 \\ &x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

(Note relationship to HW#18)

As a first step, let's introduce slack variables and replace the free variable  $x_3$  with  $x_3 - x_4$  with  $x_3, x_4 \geq 0$ . The constraints are now in the form  $Ax=b$ :

$$\begin{aligned} &\max x_3 - x_4 \text{ subject to} \\ &-3x_1 - x_2 + x_3 - x_4 + x_5 = 0 \\ &-2x_1 - 5x_2 + x_3 - x_4 + x_6 = 0 \\ &x_1 + x_2 = 1 \\ &\text{all variables} \geq 0 \end{aligned}$$

or the tableau

$$\begin{array}{ccccccc} -3 & -1 & 1 & -1 & 1 & 0 & 0 \\ -2 & -5 & 1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{array}$$

By pivoting on the 3,1-entry you get a basic feasible solution that you can use to start the simplex algorithm:

$$\begin{array}{ccccccc} 0 & 2 & 1 & -1 & 1 & 0 & 3 & 5 \\ 0 & -3 & 1 & -1 & 0 & 1 & 2 & 6 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{array}$$

The simplex algorithm produces this sequence of iterates:

$$\begin{array}{l} \text{tableau:} ((0 \ 5 \ 0 \ 0 \ 1 \ -1 \ 1) \quad \text{basis}=(5 \ 3 \ 1) \\ (0 \ -3 \ 1 \ -1 \ 0 \ 1 \ 2) \\ (1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1) \\ (0 \ 3 \ 0 \ 0 \ 0 \ -1 \ -2)) \end{array}$$

$$\begin{array}{l} \text{tableau:} ((0 \ 1 \ 0 \ 0 \ 1/5 \ -1/5 \ 1/5) \quad \text{basis}=(2 \ 3 \ 1) \\ (0 \ 0 \ 1 \ -1 \ 3/5 \ 2/5 \ 13/5) \\ (1 \ 0 \ 0 \ 0 \ -1/5 \ 1/5 \ 4/5) \\ (0 \ 0 \ 0 \ 0 \ -3/5 \ -2/5 \ -13/5)) \end{array}$$

Solve instead by using the revised simplex algorithm.

**HW 20:** on page 199-200, the first two iterations of the revised simplex algorithm are performed. Perform the third iteration.

**HW 21:** Find a "good basis" (using the method on p.207) for the following cutting stock problem:

```
raw size = 150
orders =      97 610 395 211
order sizes = 80 58 23 12
(in other words 97 of size 80 are ordered, ...)
```

**HW 22:** Solve one of the following problems by the method described in the pdf file "Cutting stock with inverse of basis". Use the "good basis" heuristic to find initial feasible basic solution. You may want to write a computer program to do the grinding for you, but that is entirely optional.

Chvátal, page 211: 13.2a, 13.2b, 13.3

Of these, the last one (13.3) is likely to involve the fewest iterations.