

(This problem copied from Chvatal, p.10, #1.7)

An oil refinery produces four types of raw gasoline: alkylate, catalytic-cracked, straight-run, and isopentane. Two important characteristics of each gasoline are its performance number PN (indicating antiknock properties) and its vapor pressure RVP (indicating volatility). These two characteristics, together with the production levels in barrels per day, are as follows:

	PN	RVP	Barrels produced
Alkylate	107	5	3,814
Catalytic-cracked	93	8	2,666
straight-run	87	4	4,016
Isopentane	108	21	1,300

These gasolines can either be sold raw at \$4.83 per barrel, or blended into aviation gasolines (Avgas A and/or Avgas B). Quality standards impose certain requirements on the aviation gasolines; these requirements, together with the selling prices are as follows:

	PN	RVP	Price per barrel
Avgas A	at least 100	at most 7	\$6.45
Avgas B	at least 91	at most 7	\$5.91

The refinery aims for the plan that yields the largest possible profit. Formulate (but do not solve) as an LP problem. Be sure to say exactly what your variables are and what each one represents.

Hints: (1) You must decide how much Alkylate, how much Catalytic-cracked, how much straight-run, and how much isopentane to blend into Avgas A, and how much of each to blend into Avgas B. (2) The PN (and likewise the RVP) of the resulting mixtures are weighted averages of the ingredients. For example, a mixture of 1000 barrels of Alkylate, 2000 barrels of Catalytic-cracked, 2000 barrels of straight-run, and 500 barrels of isopentane, has a PN of

$$\frac{1000 \times 107 + 2000 \times 93 + 2000 \times 87 + 500 \times 108}{1000 + 2000 + 2000 + 500}$$

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**solution:**

*Variable names*

	raw	avgas A	avgas B
alkylate	a_r	a_A	a_B
catalytic	c_r	c_A	c_B
straight	s_r	s_A	s_B
isopentane	i_r	i_A	i_B

*production constraints:*

$$a_r + a_A + a_B = 3814$$

$$c_r + c_A + c_B = 2666$$

$$s_r + s_A + s_B = 4016$$

$$i_r + i_A + i_B = 1300$$

**PN quality constraints:**

$$107 a_A + 93 c_A + 87 s_A + 108 i_A \geq 100 (a_A + c_A + s_A + i_A)$$

$$107 a_B + 93 c_B + 87 s_B + 108 i_B \geq 91 (a_B + c_B + s_B + i_B)$$

**RVP quality constraints:**

$$5 a_A + 8 c_A + 4 s_A + 21 i_A \leq 7 (a_A + c_A + s_A + i_A)$$

$$5 a_B + 8 c_B + 4 s_B + 21 i_B \leq 7 (a_B + c_B + s_B + i_B)$$

**nonnegativity constraints:**

all variables  $\geq 0$ .

**objective function = profit =**

$$\begin{aligned} &4.83(a_r + c_r + s_r + i_r) \\ &+ 6.45(a_A + c_A + s_A + i_A) \\ &+ 5.91(a_B + c_B + s_B + i_B) \end{aligned}$$