(p11 #1.8 in Chvátal)

An electronics company has a contract to deliver 20,000 radios within the next four weeks. The client is willing to pay

$20 for each radio delivered by the end of week 1
$18 for each radio delivered by the end of week 2
$16 for each radio delivered by the end of week 3
$14 for each radio delivered by the end of week 4

Each worker can assemble 50 radios/wk.
Size of present work force = 40.
Any experienced worker can be taken off the assembly line to instruct a class of three trainees; after one week of instruction, each of the trainees can either proceed to the assembly line or instruct additional new classes.
Weekly wages for experienced workers: $200
Weekly wages for trainees: $100
Production costs, excluding wages: $5/radio.

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solution:

Let a_i = number of assemblers in i^{th} week, i=1,2,3,4.

Let t_i = number of instructors in i^{th} week, i=1,2,3,4.

Let x_i = number of idle workers in i^{th} week, i=1,2,3,4.

profit $= 50(20a_1 + 18a_2 + 16a_3 + 14a_4)$ (revenue)

$- 50(5a_1 + 5a_2 + 5a_3 + 5a_4)$ (production costs)

$- 200 \sum_{i=1}^{4} (a_i+t_i+x_i)$ (wages, experienced workers)

$- 100 \sum_{i=1}^{4} 3t_i$ (wages, trainees)

Maximize profit subject to

$a_1+t_1+x_1 = 40$ (present work force)

$a_{i+1}+t_{i+1}+x_{i+1} = a_i+t_i+x_i+3t_i$ (i=1,2,3) (trainees become experienced)

$50(a_1+a_2+a_3+a_4) = 20,000$ (need 20,000 radios)

$t_4 = 0$ (no instructors last week)

$t_i \geq 0, \quad x_i \geq 0, \quad t_i \geq 0, \quad i=1,2,3,4.$

Remark: This is actually an integer programming problem, rather than a pure LP. The answer produced by the simplex algorithm might not be directly usable because it may require using fractional numbers of people.