Location Problems

In problems a-e, a company has factories located at the points (5,8), (-1,-13), (7,2), (3, 15), and (-8,4). It is desired to locate a new office in the best possible location. The answer depends on the meaning of "best", which has many interpretations.

a) Suppose that "best" means that the sum of the squares of the Euclidean distances from the new office to the factories is as small as possible. Use a little bit of calculus to find exactly where the new office should be placed.

b) Suppose that "best" means that the Euclidean distance from the office to the farthest factory is to be minimized. Do you think that this can be expressed as a linear programming problem?

c) Suppose we define the "distance" between any two points (x,y) and (a,b) in the plane to be $|x-a|+|y-b|$. If "best" means that the maximum distance from the office to the factories is to be minimized, can the location problem be expressed as a linear programming problem?

d) Suppose we define the "distance" between any two points (x,y) and (a,b) in the plane to be $\max\{|x-a|,|y-b|\}$. If "best" means that the maximum "distance" from the office to the factories is to be minimized, can the location problem be expressed as a linear programming problem?

e) Suppose we define "distance" in the sense of (d), but that "best" means that the *sum* of the "distances" from the office to the various factories is to be minimized, can the location problem be expressed as a linear programming problem?