Computing the inverse of the basis

Consider the problem: max c'x subject to Ax=b x ≥ 0, where b'=(13,6,17,17) and c'=(1 -2 0 3 -4 0 0) and where A is the 4×7 matrix:

0	1	2	3	4	5	6
1	**2**	1	1	2	3	2
3	1	2	**0**	1	1	1
1	4	-1	1	**2**	1	1
2	5	0	1	1	1	**0**

You can check that x = (0 3 0 1 1 0 2) is a basic feasible solution corresponding to the basis {1,3,4,6} (notice that the first column is column 0).

We remark, for future reference that the inverse of the basic matrix (the submatrix consisting of columns 1,3,4,6 is

0	1/2	-1/2	1/2
1	-2	0	0
-1	-1/2	5/2	-3/2
1	1	-2	1

The following pivot operations will put A|b into form where an identity submatrix occurs in columns 1,3,4,6:

pivot on entry in row 0, column 1
pivot on entry in row 1, column 3
pivot on entry in row 2, column 4
pivot on entry in row 3, column 6

If these same pivot operations are performed with an identity matrix tagging along, then we end up with the A_B^{-1} in the positon where the identity matrix started. If we let b tag along as well, we end up with $A_B^{-1}b$ in the position where b started.

0	1	2	3	4	5	6	b				
1	**2**	1	1	2	3	2	13	1	0	0	0
3	1	2	0	1	1	1	6	0	1	0	0
1	4	-1	1	2	1	1	17	0	0	1	0
2	5	0	1	1	1	0	17	0	0	0	1
1	-2	0	3	-4	0	0	0				

pivot on entry in row 0, column 1, to get:

1/2	1	1/2	1/2	1	3/2	1	13/2	1/2	0	0	0
5/2	0	3/2	**-1/2**	0	-1/2	0	-1/2	-1/2	1	0	0
-1	0	-3	-1	-2	-5	-3	-9	-2	0	1	0
-1/2	0	-5/2	-3/2	-4	-13/2	-5	-31/2	-5/2	0	0	1
2	0	1	4	-2	3	2	13				

pivot on entry in row 1, column 3, to get:

3	1	2	0	1	1	1	6	0	1	0	0
-5	0	-3	1	0	1	0	1	1	-2	0	0
-6	0	-6	0	**-2**	-4	-3	-8	-1	-2	1	0
-8	0	-7	0	-4	-5	-5	-14	-1	-3	0	1
22	0	13	0	-2	-1	2	9				

pivot on entry in row 2, column 4, to get:

0	1	-1	0	0	-1	-1/2	2	-1/2	0	1/2	0
-5	0	-3	1	0	1	0	1	1	-2	0	0
3	0	3	0	1	2	3/2	4	1/2	1	-1/2	0
4	0	5	0	0	3	**1**	2	1	1	-2	1
28	0	19	0	0	3	5	17				

pivot on entry in row 3, column 6, to get:

2	1	3/2	0	0	1/2	0	3	0	1/2	-1/2	1/2	1
-5	0	-3	1	0	1	0	1	1	-2	0	0	3
-3	0	-9/2	0	1	-5/2	0	1	-1	-1/2	5/2	-3/2	4
4	0	5	0	0	3	1	2	1	1	-2	1	6
8	0	-6	0	0	-12	0	7					

At this point we could start the simplex algorithm, with the starting basis {1,3,4,6}. Actually, only one pivot (on the entry in row 3 column 0) is required to finish the job:

pivot on entry in row 3, column 0, to get:

0	1	-1	0	0	-1	-1/2	2	-1/2	0	1/2	0	1
0	0	13/4	1	0	19/4	5/4	7/2	9/4	-3/4	-5/2	5/4	3
0	0	-3/4	0	1	-1/4	3/4	5/2	-1/4	1/4	1	-3/4	4
1	0	5/4	0	0	3/4	1/4	1/2	1/4	1/4	-1/2	1/4	0
0	0	-16	0	0	-18	-2	3					

Notice that to update x_B* and the inverse matrix A_B^{-1}, all you really need to know is the entering column. We could have done the same computation leaving out all the other columns. That is, starting with

0	x_B*					BASIS
2	3	0	1/2	-1/2	1/2	1
-5	1	1	-2	0	0	3
-3	1	-1	-1/2	5/2	-3/2	4
4	2	1	1	-2	1	6

we could have pivoted on the fourth entry in the entering column 0 to obtain:

0	2	-1/2	0	1/2	0	1
0	7/2	9/4	-3/4	-5/2	5/4	3
0	5/2	-1/4	1/4	1	-3/4	4
1	1/2	1/4	1/4	-1/2	1/4	0

thereby updating A_B^{-1} and x_B^* with far less work.

We don't need to do another iteration, but ignore that fact for a moment. Notice how all the information we need for the revised simplex method is available to us because of the fact that we know A_B^{-1} :

1) We can easily compute $y' = c_B' A_B^{-1}$ without solving any system of linear equations.
2) We can easily compute $d = A_B^{-1} a$ without solving any system of linear equations.