**Math. 2403, Practice Test1**

1. (a) Solve the initial value problem

$$\frac{dx}{dt} = \frac{2}{3x+1}, \quad x(0) = 3.$$

(b) Draw the associated slope field.
(c) Draw the phase portrait.

2. Solve the initial value problem

$$\frac{dx}{dt} - \frac{2x}{t} = t^2 \cos t, \quad x(\pi) = 1.$$

3. A falling object is subjected to air resistance that is proportional to the velocity of the object. Suppose that the proportionality constant is equal to $k$, the object has mass $m$, and the acceleration due to gravity is equal to $g$.
(a) Derive an equation governing the velocity $v$ of the object.
(b) Solve the differential equation and determine the limiting (or terminal) velocity of the object.

4. Find the general solution of the system

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \mathbf{x}.$$

Draw a phase portrait for the system. The origin is a critical point. Describe its type and stability.

5. Solve the initial value problem

$$2y'' - 7y' + 3y = 0, \quad y(0) = 1, \; y'(0) = -2.$$

Describe the behavior of the solution for increasing $t$.