

Math. 2403, Practice Test2

1. Use the method of undetermined coefficients or variation of parameters to solve the initial value problem

$$y'' - y = -4xe^{-x}, \quad y(0) = 0, y'(0) = 1.$$

2. An 8-kg mass is attached to a spring hanging from the ceiling, thereby causing the spring to stretch 1.96 m upon coming to rest at equilibrium. At time $t = 0$, an external force $f(t) = \cos 2t$ N is applied to the system. The damping constant for the system is 3 N-sec/m. Determine the steady state solution of the system (i.e. the part of the solution that is left for big t after the transient solution has been eliminated.) Assume that $g = 9.8$ m/sec².

3.(a) Is the function

$$f(t) = e^{\frac{t^2}{t+1}}.$$

of exponential order α for some α ? If yes, find the range of α .

(b) Find $L\{f(t)\}$ if

$$f(t) = \begin{cases} e^{2t} & \text{if } 0 < t < 3 \\ 1 & \text{if } t > 3. \end{cases}$$

4. Find $L^{-1}\{F(s)\}$ if

$$F(s) = \frac{3s + 5}{s^2 - 2s + 5}.$$

5. Use Laplace transform to solve the initial value problem

$$y''' + y'' + y' + y = 1, \quad y(0) = y'(0) = 0, y''(0) = 1.$$