MATH 4032 (Spring'13) – Supplementary Problems

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Office Hours: **Wed. Fri. 1:00-2:00pm, Thurs. 2:00–3:00pm**

For Practice Only: no need to submit

I. Chapter 3 of Cameroon's textbook: Exercises 9, 11, 14, 15 from Section 3.13.

II. Chapter 4 of Cameroon's textbook: Exercises 13, 14, 15, 16, 18 from Section 4.8.

III. Recall Property B problem: Let $m(n)$ = the smallest number of n-sets over some universe of elements which are not properly 2-colorable – meaning that there is no way to assign Red/Blue to the elements in the universe, without making some n-set monochromatic. Show that $m(n) \geq 2^{n-1}$.

Hint: Use the probabilistic method.

IV. There are k people in an elevator at the ground floor. Each wants to get out at a random floor of one of the n upper floors. What is the expected number of stops by the elevator?

Hint: The answer is $n[1-(1-1/n)^k]$.

V. A *dominating set* D in a graph $G = (V, E)$ is a subset of vertices so that each vertex either belongs to D or is adjacent to some vertex in D.

(i) Observe that for any subset $S \subseteq V$, $D := S \cup (V \setminus N(S))$ is always a dominating set. (Here, by $N(S)$ we mean the union of neighborhoods of vertices in S.)

(ii) Let G be a graph on n vertices and minimum degree, $\min_u d(u) = \delta$. Show that there is a dominating set of size at most $n[1+\ln(1+\delta)]/(1+\delta)$.

Hint: Use the probabilistic method, with an *alteration*: Choose a set S randomly, by placing each vertex of V with probability p, independently of other vertices. Estimate $E[|S|+|V \setminus N(S)|]$ in terms of n and p. Optimize with respect to p. (You might find it convenient to use the inequality, $1-x \leq e^{-x}$, for x: real.)

Reminder. Test 1 on Monday, Feb. 11th, in class. OPEN NOTES, but no textbooks allowed.