Test 3, Version A
Math 1501, Fall 02.

Prof. W. Gangbo *

October 28, 2002

Student Name: **Student Section:**

Teaching Assistant Name:

Instructions. You are to work independently these exercises for the next fourty five minutes (45 mn.). You may not use any textbook or your class notes during the text. Read carefully each exercise and show all your work.

*School of Mathematics, Georgia Institute of technology.

Exercise 1 (**25 points**) Assume that

$$f(x) = \begin{cases} 2 - 2x - x^2 & \text{if} \quad -2 \leq x \leq 0 \\ |x - 2| & \text{if} \quad 0 < x < 3 \\ \frac{1}{3}(x-2)^3 & \text{if} \quad 3 \leq x \leq 4. \end{cases}$$

(a) (**10 points**) Find the critical points of f.

(b) (**10 points**) Find the local extreme values of f.

(c) (**5 points**) Classify the local extreme values of f.

Exercise 2 (**25 points**) Set

$$f(x) = 2\cos^2 x - x^2, \qquad x \in [0, \pi].$$

a) (**5 points**) Evaluate f' and f".

b) (**15 points**) Find the points of inflection of f.

c) (**5 points**) Describe the concavity of f.

Exercise 3 (**20 points**) Determine whether or not the graph of $x \to f(x) = 2x^{\frac{3}{5}} - x^{\frac{6}{5}}$ has a vertical tangent or a vertical cusp at $c = 0$. Justify your answer.

Exercise 4 (**30 points**) Assume that

$$f(x) = 3x^2, \qquad x \in [0, 5^{\frac{1}{3}}].$$

a) (**5 points**) For $P = \{x_0, x_1, \cdots, x_n\}$, an arbitrary partition of $[0, 2]$, write $L_f(P)$ and $U_f(P)$, the lower sum and the upper sum of f.

b) (**5 points**) For each $i = 1, 2, \cdots, n$ consider the three numbers x_i^2, x_{i-1}^2, and $\frac{1}{3}(x_{i-1}^2 + x_{i-1}x_i + x_i^2)$. Which one of them is the largest and which one is the smaller?

c) (**10 points**) Denote the following sum by $M_f(P)$:

$$(x_1-x_0)\frac{x_0^2+x_0x_1+x_1^2}{3}+(x_2-x_1)\frac{x_1^2+x_1x_2+x_2^2}{3}+\cdots+(x_n-x_{n-1})\frac{x_{n-1}^2+x_{n-1}x_n+x_n^2}{3}.$$

Use b) to deduce which one of the numbers $L_f(P)$, $U_f(P)$, and $M_f(P)$ is the largest, and which one is the smaller one.

d) (**5 points**) For each $i = 1, 2, \cdots, n$ say which one of the two numbers

$$(x_i - x_{i-1})\frac{x_{i-1}^2+x_{i-1}x_i+x_i^2}{3}, \qquad \frac{x_i^3-x_{i-1}^3}{3}$$

is the smaller.

e) (**5 points**) Use a), c) and d) to evaluate $\int_0^{5^{\frac{1}{3}}} f(x)dx$.