

Test 4, Version A  
Math 1501, Fall 02.

Prof. W. Gangbo \*

November 14, 2003

**Student Name:**

**Student Section:**

**Teaching Assistant Name:**

**Instructions.** You are to work independently these exercises for the next forty five minutes (45 mn.). You may not use any textbook or your class notes during the text. Read carefully each exercise and show all your work.

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**Exercise 1 (15 points)** Calculate the derivative of the function  $x \rightarrow f(x) = \int_1^{2x+1} \frac{1}{2} \sec u \tan u \, du$ .

**Exercise 2 (25 points)** Let

$$f(x) = 2 \cos x + \sin x, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

a) **(5 points)** Evaluate  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) dx$ .

b) **(20 points)** Find the area of the region bounded by the graph of  $f$  and the  $x$ -axis for  $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ . (You may use that  $\sin y = \frac{\tan y}{\sqrt{1+\tan^2 y}}$  and  $\cos y = \frac{1}{\sqrt{1+\tan^2 y}}$ ).

**Exercise 3 (15 points)** Use  $u$ -substitution to evaluate  $\int \frac{b^3 x^3}{\sqrt{1-a^4 x^4}} dx$ .

**Exercise 4 (20 points)** Find the area of the bounded region determined by the curves

$$y = \sin x, \quad y = \sin 2x, \quad x \in [0, \pi/2].$$

**Exercise 5 (25 points)** Find the first coordinate  $\bar{x}$  of the centroid  $(\bar{x}, \bar{y})$  of the bounded region determined by the curves

$$y = 6x - x^2, \quad x + y = 6.$$