

Final Exam, Version A  
Math 1501, Fall 02.

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December 7, 2002

**Student Name:**

**Student Section:**

**Teaching Assistant Name:**

**Instructions.** You are to work independently these exercises for the next two hours (2h 00mn.). You may not use any textbook or your class notes during the text. Read carefully each exercise and show all your work.

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**Exercise 1 (15 points)** Find the largest possible area for a rectangle inscribed in a circle of radius 4.

**Exercise 2 (10 points)** Assume that  $f$  is a continuous function and that

$$\int_0^x tf(t)dt = \sin x - x \cos x.$$

Determine  $f(\frac{\pi}{2})$  and find  $f'(x)$ .

**Exercise 3 (15 points)** The region between the graph of  $y = \sin x$  and the  $x$ -axis,  $0 \leq x \leq \pi$  is revolved around the line  $y = 1$ . Find the volume of the solid that is generated.

**Exercise 4 (15 points)** Let  $f$  be defined by

$$f(x) = \int_1^{2x} \sqrt{16 + t^4} dt, \quad x \in \mathbb{R}.$$

(a) **(5 points)** Find the range of  $f$ . Justify your answer.

b) **(5 points)** Say whether or not  $f$  has an inverse, and justify your answer.

(c) (**5 points**) If  $f$  has not inverse, you need not to do anything. If  $f$  has an inverse, say whether or not  $f^{-1}$  is differentiable at 0. Evaluate  $(f^{-1})'(0)$  in case it exists, otherwise you need not to do anything.

**Exercise 5 (15 points)** Define  $f(x) = x^{\sin x}$ .

(a) **(10 points)** Find the domain of definition of  $f$ , find the set where  $f$  is differentiable and determine  $f'(x)$ .

b) **(5 points)** Say whether or not  $\lim_{x \rightarrow +\infty} f(x)$  exists. Justify your answer.

**Exercise 6 (15 points)** Evaluate  $I = \int_0^1 x 10^{1+x^2} dx$ .

**Exercise 7 (15 points)** Integrating by parts twice, evaluate  $F(x) = \int \cos(\ln x) dx$ .  
Hint: First, use a  $u$ -substitution to express differently  $F(x)$ .