MATH 1501 Sample Quiz Questions for Test 1, Fall 2007 WTT

Note 1: There are approximately two to three times as many problems listed here as you can expect on an hour exam, but this more comprehensive version should be of greater assistance to students in studying for the test. Also, in some cases, I haven't left as much space on this page as will be the case on an actual test.

1. The statement $\lim_{x \to c} f(x) = L$ means:

2. The statement $f(x)$ is continuous at c means:

3. Sketch the graph of a function f which is:

a. continuous for all values of x except $x = 1$.

b. continuous from the left at $x = 1$;

c. not continuous from the right at $x = 1$;

d. not differentiable at $x = 2$ and at $x = 4$.

4. Given $f(x) = \sin x$ and $g(x) = \cos x$, sketch the graphs of

a. $2f + 3g$.

b. fg.

c. $f \circ g$.

5. Give examples of functions f and g so that

a. For every real number c, neither f nor g is continuous at c;

b. for every real number c, the sum $f + g$ is continuous at c;

c. for every real number c, the product fg is continuous at c;

(Hint: Consider functions defined one way on rationals and another on irrationals).

6. The derivative f' of a function f is defined by $f'(x) =$

7. Use your answer to the preceding problem to determine the derivative f' of the function f when (Note: On an hour test, you would expect only one such problem):

a. $f(x) = mx + b$.

b. $f(x) = 5x^2 - 7x + 13$.

c. $f(x) = \sqrt{8x - 23}$.

d. $f(x) = 1/x^2$.

8. State the following theorems and draw an appropriate graph to illustrate the concept (Note: Again, on an hour test, you would expect only one such problem. Similar remarks apply to many of the problems to follow).

a. The Intermediate Value Theorem.

b. The Extreme Value Theorem.

c. Rolle's Theorem.

d. The Mean Value Theorem

9. Use derivative formulas to find $f'(x)$ when

a. $f(x) = 8x^5 - 17x^3 + 12x^2 + 23$.

b. $f(x) = \sqrt{\sin 8x}$.

c. $f(x) = \cos(\sqrt{8x})$.

d. $f(x) = (\tan^4 x)/\sqrt{6x}$.

10. A baseball is thrown at an initial velocity of 160 feet per second at an angle which is 30 degrees above horizontal. Assuming that it is released 6 feet above ground,

a. How far above ground will the baseball go?

b. How long will the ball be in the air?

c. How far away from where it is thrown will it land?

11. A 25 foot long ladder is leaning against a wall so that the bottom of the ladder is 5 feet away from the wall. A painter walks by and inadvertently snags the bottom of the ladder dragging it away from the wall at a rate of 3 feet per second. The top of the ladder just slides down the wall. How fast is the top of the ladder moving (towards the ground) when it is 10 feet above the ground?

12. Consider a function f whose derivative f' is defined by

$$f'(x) = 3x^2(x-2)^3(x+5)(x+2)^4$$

Find

a. The intervals on which f is increasing.

b. The intervals on which f is decreasing.

c. The values at which f achieves a local max.

d. The values at which f achieves a local min.

13. Sketch the graph of the inverse sine function $\mathrm{Sin}^{-1}\,x$ whose domain is $[-1,1]$ and whose range is $[-\frac{\pi}{2},\frac{\pi}{2}]$. Use the chain rule on the equation $\mathrm{Sin}\,x \circ \mathrm{Sin}^{-1}\,x = x$ to show that the derivative of $\mathrm{Sin}^{-1}\,x$ is $1/\sqrt{1-x^2}$.

14. If $w = 3z^2 + 2z - 5$ and $z = 4x - 1$, use the chain rule to find $\frac{dw}{dx}$ when $x = 0$.

15. A tank has the form of an inverted cone of height 62 inches with a circular opening of radius 28 inches at the top. The volume V of a cone whose base is a circle of radius r and whose height is h is given by the formula $V = \frac{1}{3}\pi r^2 h$. The tank is being filled with water at the rate of 5 cubic inches per second. How fast is the water rising in the cone when it has been filled to a depth of 40 inches?

16. Verify the conclusion of the Mean Value Theorem for the function $f(x) = x^2 - 3x + 5$ over the interval $[0,1]$.

17. Find the critical points and the local extreme values for the function f where

a. $f(x) = (1-x)(1+x)^3$.

b. $f(x) = x^3/(1+x)$.

c. $f(x) = x + \cos x$.

18. Find the volume of the largest right circular cone that fits inside a sphere of radius r.

19. Consider the design of cylindrical oil can which will contain .01 cubic meter of oil. The can will have a metal top and bottom made from aluminum costing three dollars per square meter, while the side will be made from a single piece of reinforced plastic costing two dollars per square meter. The side will be cut as a rectangular piece and then rolled into a hollow cylinder. What dimensions of the oil can minimize the total cost of materials? A newly hired engineer proposes new materials with the metal top costing \$2.05 and the side piece costing \$1.95 per square meter. Should the new employee be given a raise or given the boot?

20. Describe the concavity of the function f where

a. $f(x) = x^3(1-x)$.

b. $f(x) = x\sqrt{4-x^2}$.

21. Find all asymptotes for the function $f(x) = (6x^2 - 9x + 3)/(4x - 5)$.

22. Determine whether the function f has a vertical tangent or a cusp at the point c.

a. $f(x) = (x-5)^{7/5}; \quad c = 5$.

b. $f(x) = x(x-1)^{1/3}; \quad c = 1$.