KEY

Student Name and ID Number

MATH 1501 Test 2, October 25, 2007, WTT

1. Complete the following to form a statement of the First Fundamental Theorem: Let f and G be continuous on $[a, b]$ with G differentiable on (a, b).

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If $G'(x) = f(x)$ for all x from (a,b), then

$$\int_a^b f(t)\,dt = G(b) - G(a)$$

2. Complete the following to form a statement of the Second Fundamental Theorem: Let f be continuous on $[a, b]$ and let c be any number in $[a, b]$. Also, let F be the function defined on $[a, b]$.

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by setting $F(x) = \int_a^x f(t)\,dt$. Then

$F'(x) = f(x)$ for all x from (a,b)

3. Use differentials to estimate $\sqrt{64.03}$.

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$f(x) = \sqrt{x}$ $\qquad f(64) = 8$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\qquad f'(64) = \frac{1}{2 \cdot 8} = \frac{1}{16}$

$f(64.03) = \sqrt{64.03} \approx \boxed{8 + \frac{1}{16} \cdot .03}$

4. The Newton-Raphson method uses the formula $x_{n+1} = x_n - f(x_n)/f'(x_n)$. If $f(x) = x^2 - 35$ and $x_2 = 5.0$, find x_3.

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$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 5.0 - \frac{25-35}{2 \cdot 5} = 5 - \frac{-10}{10}$$

$$= 5 + 1 = \boxed{6}$$

5. Consider the function $f(x) = x^2$, the closed interval $[1, 10]$ and the partition $P = [1, 2, 7, 10]$.

a. What is the mesh $\mu(P)$ of the partition P?

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$\mu(P) = \max \Delta x_i = \boxed{5}$

b. For the selection $t_1 = 2$, $t_2 = 2$, and $t_3 = 10$, what is the value of the Reimann sum $\Sigma_{i=1}^{n} f(t_i)\Delta(x_i)$?

$$2^2 \cdot 1 + 2^2 \cdot 5 + 10^2 \cdot 3 = 4 + 20 + 300 = \boxed{324}$$

6. Complete the following to make correct statements.

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- The natural logarithm function $\ln x$ is defined by $\ln x =$ $\boxed{\int_1^x \frac{1}{t}\,dt}$
- The derivative of $\ln x$ is $\boxed{\frac{1}{x}}$
- $\ln x$ is strictly increasing because $\boxed{\frac{1}{x} > 0 \text{ when } x > 0}$
- The domain of $\ln x$ is $\boxed{x > 0}$ or $\boxed{(0, \infty)}$

One point each question

- The range of $\ln x$ is $(-\infty, \infty)$ or $\mathbb{R}$ or all real numbers
- If x and y are positive, then $\ln(xy) =$ $\ln x + \ln y$
- If x is positive and q is rational, then $\ln(x^q) =$ $q \ln x$
- The exponential function $\mathrm{Exp}(x)$ is defined as compositional inverse of $\ln x$
- The domain of $\mathrm{Exp}(x)$ is $(-\infty, \infty)$ or $\mathbb{R}$ or all real numbers
- The range of $\mathrm{Exp}(x)$ is $(0, \infty)$ or all positive real numbers
- The derivative of $\mathrm{Exp}(x)$ is $\mathrm{Exp}(x)$
- $\mathrm{Exp}(x)$ is strictly increasing because $\mathrm{Exp}(x) > 0$ for all x
- The number e is unique solution to $\ln x = 1$
- If q is rational, then $e^q = \mathrm{Exp}(q)$ because $\ln(e^q) = q \ln e = q \cdot 1 = q = \ln \mathrm{Exp}(q)$
- For all irrational values of x, we then set $e^x =$ $\mathrm{Exp}(x)$
- For all x, y, $e^x e^y =$ e^{x+y}
- For all x, y, $(e^x)^y =$ e^{xy}
- When a is positive and x is irrational, we define $a^x =$ $e^{x \ln a}$
- The function $\mathrm{AT}(x)$ (which will eventually be called the inverse tangent or arctangent function) is defined by $\mathrm{AT}(x) =$ $\int_0^x \frac{1}{1+t^2}\,dt$
- The domain of $\mathrm{AT}(x)$ is $(-\infty, \infty)$ or $\mathbb{R}$ or all real numbers
- The derivative of $\mathrm{AT}(x)$ is $\frac{1}{1+x^2}$
- $\mathrm{AT}(x)$ is strictly increasing because derivative is always positive
- The range of $\mathrm{AT}(x)$ is an open interval of the form $(-b, b)$. In fact $b =$ $\pi/2$

7. Define a function h by

$$h(x) = \int_{\pi/2}^{x} \sinh s \cos 3s\, e^{3s}\, ds$$

Find $h'(x)$

$$h'(x) = \sinh x \cos 3x\, e^{3x}$$

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8. Find the derivatives of the following functions:

a. $f(x) = \ln \sinh(e^x + \frac{1}{x})$

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$$f'(x) = \frac{1}{\sinh(e^x + \frac{1}{x})} \cdot \cosh\left(e^x + \frac{1}{x}\right)\left(e^x - \frac{1}{x^2}\right)$$

5 pts each part

b. $g(x) = x^{\sin x}$ $= e^{(\sin x)\ln x}$

$$g'(x) = e^{(\sin x)\ln x}\left[(\sin x)\frac{1}{x} + (\ln x)\cos x\right]$$

9. Calculate;

a. $\int e^u(1+e^u)^{-7/5}\,dy$ $= \dfrac{(1+e^u)^{-2/5}}{-2/5} + C = \boxed{-\frac{5}{2}(1+e^u)^{-2/5} + C}$

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5 pts each part

b. $\int_0^\pi x^2 e^{x^3}\,dx$ $= \frac{1}{3}\int_0^\pi 3x^2 e^{x^3}\,dx = \frac{1}{3}e^{x^3}\Big|_0^\pi = \boxed{\frac{1}{3}\left(e^{\pi^3} - 1\right)}$

10. A radio-active substance of quantity 36 grams reduces to 12 grams after 2 years. Express its half-life in years.

$A = A_0 e^{-kt}$ where $A_0 = 36$

$12 = 36e^{-k\cdot 2}$

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$e^{2k} = 3$

$2k = \ln 3$

$k = \dfrac{\ln 3}{2}$

Now lets find t so that

$18 = 36e^{-\frac{\ln 3}{2}t}$

$e^{\frac{\ln 3}{2}t} = 2$

$\frac{\ln 3}{2}t = \ln 2$

$$\boxed{t = \frac{2\ln 2}{\ln 3}}$$

GRADING SUMMARY
99 total pts, so
blank paper gets +1.