**Math 1502 Quiz3 Summer 2007**

**Solutions for section B1, B2**

**Problem 1**
Find the general solution of $y' - 2y = e^x$.
**Solution:** This is a 1st order linear equation with standard form $y' + P(x)y = Q(x)$.
Here $P(x) = -2$, $Q(x) = e^x$, so the integrating factor $H(x) = \int P(x)dx = \int -2dx = -2x$.
The general solution is

$$\begin{aligned} y(x) &= e^{-H(x)}(\textstyle\int e^{H(x)}Q(x)dx + C) \\ &= e^{2x}(\textstyle\int e^{-2x}e^x dx + C) \\ &= e^{2x}(-e^x + C) \end{aligned}$$

**Problem 2**
Solve the initial value problem: $x^2y' = y - xy$, $y(1) = 1$.
**Solution:** This is a separable equation.

$$\frac{dy}{dx} = y' = \frac{y(1-x)}{x^2}$$

so

$$\frac{dy}{y} = \frac{(1-x)}{x^2}dx$$

take the integral on both sides:

$$\begin{aligned} ln(y) &= \textstyle\int \frac{(1-x)}{x^2}dx + C \\ &= \textstyle\int \frac{1}{x^2} - \frac{1}{x}dx + C \\ &= -x^{-1} - ln(x) + C \end{aligned}$$

Plug in the initial value $y(1) = 1$ to calculate the constant $C$:

$$ln(1) = -1 - ln(1) + C \Longrightarrow C = 1.$$

Thus $ln(y) = -x^{-1} - ln(x) + 1$.