Student Name and ID Number

MATH 3012, Quiz 3, November 22, 2011, WTT

1. Find the general solution to the advancement operator equation:
$A^2(A-4)^3(A+6)^4(A-5)f(n) = 0$

2. Find the solution to the advancement operator equation:
$(A^2 - 8A + 12)f(n) = 0, \quad f(0) = 1$ and $f(1) = 34$.

3. Find a particular solution to the advancement operator equation:
$(A+3)g(n) = 7 \cdot 3^n$.

4. Find the form of a particular solution to (Express your answer with constants to be determined. Do not attempt to carry out the solution):
$(A-4)^3(A+6)^4(A-5)f(n) = 7 \cdot 5^n + 3n$.

5. How many permutations of $\{1, 2, 3, \ldots, 23\}$ satisfy the four requirements: $\sigma(3) = 3$, $\sigma(4) = 4$, $\sigma(11) = 11$ and $\sigma(19) = 19$?

6. How many functions from $\{1, 2, 3, \ldots, 18\}$ to $\{1, 2, 3, \ldots, 11\}$ satisfy the three requirements: 2 is not in the range, 6 is not in the range, and 8 is not in the range.

7. The integer $65,000$ can be factored as $2^3 \cdot 5^4 \cdot 13$. Use the inclusion-exclusion formula to find $\phi(65,000)$.

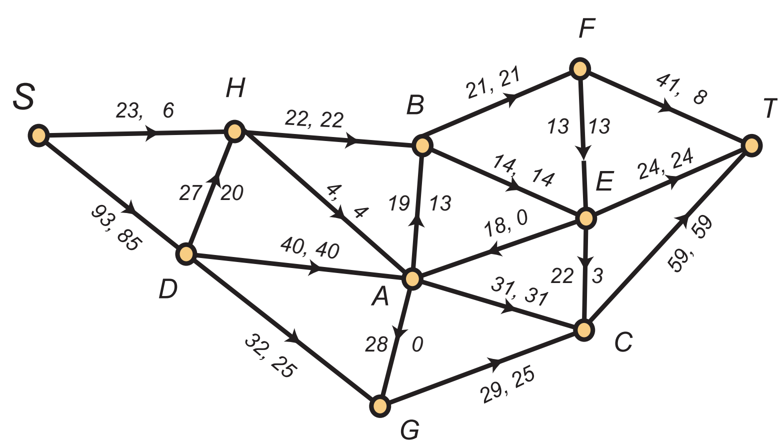
8. Interpret the coefficients of the function $(1+x)(1+x^3)(1+x^7)/(1-x^2)$ in terms of partitions of an integer. Then write all the partitions of the integer 11 that correspond to this interpretation.

9. Consider the data file (shown on the left below) for the weights on the edges of a graph with vertex set $\{a, b, c, d, e, f, g, h\}$. In the space to the right, list in order the edges that would be selected in carrying out Kruskal's algorithm (avoid cycles) and Prim's algorithm to find a minimum weight spanning tree. For Prim, use vertex a as the root.

graphdata.txt **Kruskal** **Prim**

e	h	10
c	e	12
b	c	13
c	d	14
b	h	15
f	g	16
a	d	17
a	b	18
b	g	20
e	d	21
a	h	32

10. Consider the following network



a. What is the current value of the flow?

b. What is the capacity of the cut $\{S, A, D, G, H\} \cup \{B, C, E, F, T\}$?

c. Write below the labels that are applied by carrying out the Ford-Fulkerson labeling algorithm.

d. Write the sequence of vertices that forms an augmenting path, as determined by the labeling done in the previous step.

e. Use the information gleaned from the previous two parts to update the flow. You may provide your answer by writing directly on the figure.

f. What is the new value value of the flow?

g. Write below the labels that are applied by carrying out the Ford-Fulkerson labeling algorithm on the updated network. It should terminate without the sink being labeled.

h. Find a cut whose capacity is the value of the current flow.