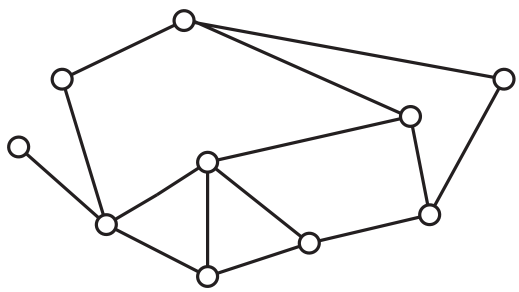
Student Name and ID Number

MATH 3012 Quiz 2, October 22, 2015

1.

(a) In the space to the right, verify Euler's formula for the graph G shown below.



(b) Explain why $\omega(G) \geq 3$.

(c) Show that $\chi(G) \leq 3$ by providing a proper coloring (write directly on the figure).

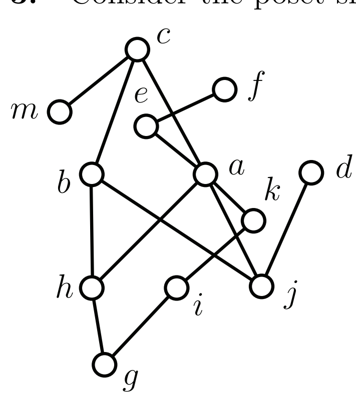
(d) Explain why $\chi(G) = \omega(G) = 3$.

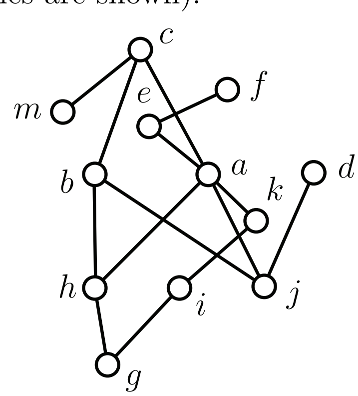
2.

(a) Complete the following sentence to form a correct definition: A graph G is *perfect* when

(b) Explain why the graph in Problem 1 above is *not* perfect.

3. Consider the poset shown below (two copies are shown).





(a) Find the set of maximal elements.

(b) Find the set of minimal elements.

(c) Find all points comparable with b. ____________

(d) Find all points incomparable with a. ____________

(e) Find all points covered by b. ____________

(f) Find all points which cover a. ____________

(g) Find a maximal chain of size 4. ____________

(h) Find a maximal antichain of size 3 containing d and g. ____________

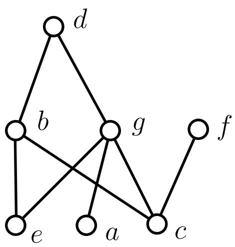
(i) Recursively strip off the minimal elements and find the height h of the poset. Also find a partition of the poset into h antichains. You may provide your answer by labelling the points in the figure on the left with integers from $\{1, 2, \ldots, h\}$ so that all points labelled with the same integer form an antichain.

The height h is ______ and ____________ is a maximum chain.

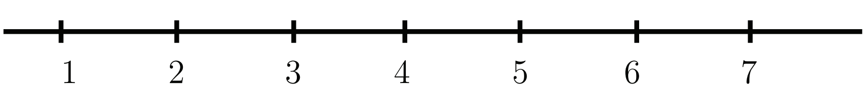
(j) Find, by inspection, the width w of the poset. Also, find a partition of the poset into w chains. You may provide your answer by labelling the points in the figure on the right with integers from $\{1, 2, \ldots, w\}$ so that points labelled with the same integer form a chain.

The width w is ______ and ____________ is a maximum antichain.

4. Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation by computing the down-sets and up-sets in the space provided. Then use the First Fit coloring algorithm to find the width w and a partition of the poset into w chains. Also, find a maximum antichain.



$D(a) =$	$U(a) =$
$D(b) =$	$U(b) =$
$D(c) =$	$U(c) =$
$D(d) =$	$U(d) =$
$D(e) =$	$U(e) =$
$D(f) =$	$U(f) =$
$D(g) =$	$U(g) =$

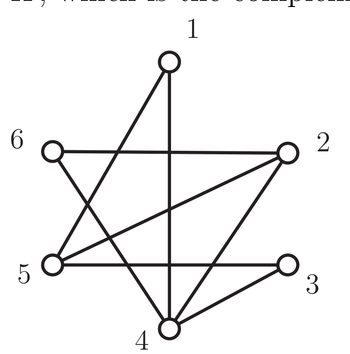


The width w is ________ and ____________ is a maximum antichain.

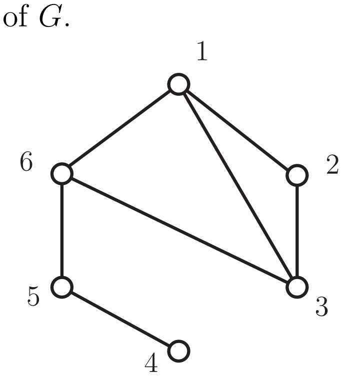
5. Draw an order diagram for the following poset: $X = \{1,2,3,4,5,6,7\}$ and $P = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(7,4),(6,1),(2,4),(3,4),(5,4),(5,2),(5,3),(6,2),(6,4)\}$.

6. Draw the order diagrams of two posets P and Q whose cover graph is the cycle C_6 so that (1) the height of P is 2 and the width of P is 3; and (2) the height of Q is 4 and the width of Q is 2.

7. Shown below on the left is a graph G with vertex set $\{1,2,3,4,5,6\}$. On the right is a graph H, which is the complement of G.



G



H

(a) Apply the algorithm we learned in class to find a transitive orientation of the graph H. You may write directly on the drawing of H. Then draw the order diagram of the poset associated with the orientation you determine.

(b) By inspection, find four points in this poset which form a copy of $\mathbf{2}+\mathbf{2}$: ________________.

(c) What conclusion concerning the graph G are you able to make on the basis of your work in parts (a) and (b)?

8. True–False. Mark in the left margin.

1. There is a planar graph 1024 vertices and 5892 edges.
2. There is a graph G with 1024 vertices and 5892 edges such that $\chi(G) = 2$.
3. There is a perfect graph with 1024 vertices and 5892 edges.
4. There is a poset with 1024 elements having height 47 and width 57.
5. When a graph G is a cover graph, there is only one poset P whose cover graph is G.
6. When a graph G is a comparability graph, there is only one poset P whose comparability graph is G.
7. The Euler formula for comparable posets having a transitive bijection mapping cover graphs to complete NP chains has a certificate that can be exchanged at Publix for an ice cream cone.