Student Name and ID Number

MATH 3012 Quiz 1, September 21, 2017, WTT

1. Consider the 52-element set consisting of the upper and lower case letters of the English alphabet.

a. How many strings of length 16 can be formed if repetition of symbols is *not* permitted?

b. How many strings of length 16 can be formed if repetition of symbols is permitted?

c. How many strings of length 16 can be formed using exactly three x's, four B's and nine A's?

d. How many strings of length 16 can be formed using exactly three x's, four B's and nine A's, with the four B's occuring consecutively?

2. How many lattice paths from $(0,0)$ to $(12,15)$ pass through $(4,9)$?

3. A wealthy donor intends to donate $1,000,000 to Georgia Tech by partitioning the total into four awards which will then be distributed to the Schools of (1) Mathematics, (2) Computer Science, (3) Industrial Science and Engineering and (4) Electrical Engineering. Each of the schools will receive an amount which is a positive multiple of $10,000.

a. In how many different ways can the donor distribute this sum?

b. In how many different ways can the donor distribute this sum if the School of Mathematics will receive at least $300,000?

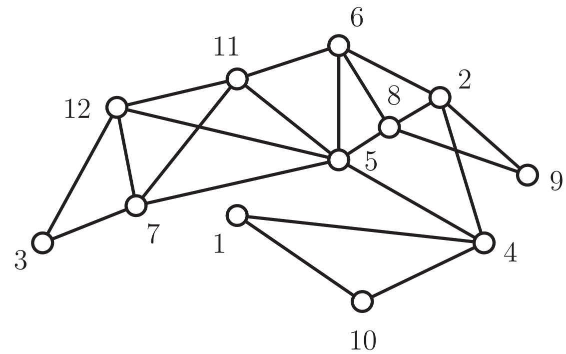
c. In how many different ways can the donor distribute this sum if the School of Mathematics will receive at least $300,000 and at most $500,000

4. Use the Euclidean algorithm to find $d = \gcd(306, 1190)$.

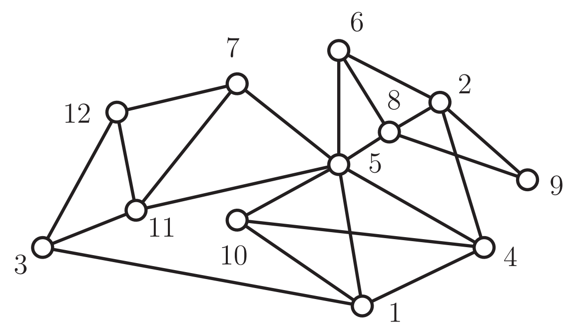
5. Use your work in the preceding problem to find integers a and b so that $d = 306a + 1190b$.

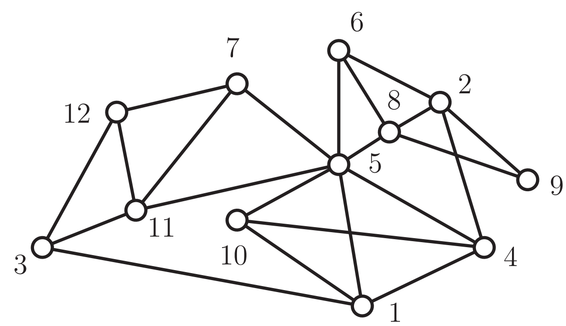
6. For a positive integer n, let t_n count the number of ternary strings of length n that do not contain 02 or 201 as a substring. Note that $t_1 = 3$, $t_2 = 8$, $t_3 = 20$. Develop a recurrence relation for t_n and use it to compute t_4 and t_5.

7. Use the greedy algorithm developed in class (always proceed to the lowest legal vertex) to find an Euler circuit in the graph G shown below (use node 1 as root):



8. Two copies of a graph G are shown below.





(a) Find a clique of size 4. ________________

(b) Find an induced cycle of size 5. ________________

(c) Show that $\chi(G) \leq 4$ by producing a proper coloring using the elements of $\{1, 2, 3, 4\}$ as colors. You may write directly on the left copy of the graph.

(d) Show that the graph has a Hamiltonian cycle starting with $(1, 3, 12, 11)$. You may either darken the appropriate edges on the second copy, or write out a suitable permutation of the vertex set.

9. True–False. Mark in the left margin.

1. $P(10, 3) = 760$.
2. $C(10, 3) = 120$.
3. Any connected graph with an even number of edges has an Euler circuit.
4. Any sequence of 37 distinct real numbers either contains an increasing subsequence of length 5 or a decreasing subsequence of length 10.
5. There is a connected graph with 100 vertices and 100 edges which does not have a Hamiltonian cycle.
6. If G is a graph on 20 vertices and every vertex has a least 12 neighbors, then G has a Hamiltonian cycle.
7. The number of lattice paths from $(0, 0)$ to $(12, 12)$ which do not go above the diagonal is the Catalan number $\binom{12}{6}/7$.
8. If G is a graph and $\chi(G) = 3$, then $\omega(G) = 3$.
9. $\log n = O(\sqrt{n})$.
10. $\log n = o(\sqrt{n})$.
11. $587n^{587} = o(2^n)$.
12. $2^n = O(587n^{587})$.
13. Connected tilings of recursive multinomial graphs admit Euler circuits with vertices having clique size larger than their chromatic number, except when they have Hamiltonian certificates whose correctness can be estimated in polynomial time by a biased referee.